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Parametric quintic spline for time fractional Burger's and coupled Burgers' equations

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Abstract

In this paper, the numerical solutions of time fractional Burger's and coupled Burgers' equations are obtained using the parametric quintic spline method with a local truncation error of order eight in distance direction. Additionally, the von Neumann method was utilized for studying the stability analysis of the present method. Finally, to show the accuracy of this method, some examples with different cases for Burger's and coupled Burgers' equations are presented and their results are compared with the previous methods.

Keywords: Parametric quintic spline method; Fractional Burger's equation; Fractional coupled Burger's equations; Von Neumann stability analysis

1 Introduction

Consider the time fractional Burger's equation (TFBE) [1–17] and the time fractional coupled Burgers' equations (TFCBEs) [18–25] defined as follows, respectively:

$$D_t^\alpha u(x,t) + u(x,t)D_x u(x,t) - sD_{xx}u(x,t) = f(x,t) \quad \text{for } 0 < \alpha < 1, \tag{1}$$

$$D_t^\alpha \big(u(x,t)\big) = D_{xx}\big(u(x,t)\big) + 2u(x,t)D_x\big(u(x,t)\big) - D_x\big(u(x,t)v(x,t)\big), \tag{2}$$

$$D_t^\beta \big(v(x,t)\big) = D_{xx}\big(v(x,t)\big) + 2v(x,t)D_x\big(v(x,t)\big) - D_x\big(u(x,t)v(x,t)\big), \tag{3}$$

where s is a viscosity parameter, $D_t^\alpha u(x,t)$ and $D_t^\beta v(x,t)$ are the Caputo fractional derivatives of orders α and β [4, 7–11, 15, 26–30] defined as follows:

$$D_t^\alpha u(x,t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\partial u(x,\xi)}{\partial \xi}(t-\xi)^{-\alpha}\, d\xi \quad \text{for } 0 < \alpha < 1, \tag{4}$$

by the same manner, we can define $D_t^\beta v(x,t)$.

Firstly, many numerical techniques are used to obtain the solutions of equations (1)–(3) such as Adomian decomposition method (ADM) [1], variational iteration method (VIM) [2], cubic parametric spline (CPS) method [3], quadratic B-spline Galerkin method (QBSGM) [4], cubic trigonometric B-splines method (CTBSM) [7], Legendre–Galerkin spectral method (LGSM) [9], Crank–Nicolson approach (CNA) [14], finite difference method (FDM) [16], Chebyshev collocation method (CCM) [20], spectral collocation